CHAPTER 10: HYPOTHESIS TESTS REGARDING A PARAMETER

[This chapter is based on Chapter 10 of the textbook]

In Chapters 8 and 9 we learned how to estimate parameters using data from a sample. In this chapter, we learn hypothesis testing, that is, how to test statements regarding a characteristic of a population. We start learning the language of hypothesis testing.

10.1 The language of hypothesis testing

Let's start with a definition.

Definition 10.1. *Hypothesis testing is a procedure, based on sample evidence and probability, used to test statements regarding a characteristic of one or more populations. The basic steps in conducting a hypothesis test are:*

(1) Make a statement regarding the nature of the population

(2) Collect evidence (data) to test the statement

(3) Analyze the data to assess the plausibility of the statement

Let's see an example.

Example 10.1. *A friend of yours wants to play a simple coin-flipping game. If the coin comes up heads, you win; if it comes tails, your friend wins. Suppose the outcome of 5 plays of the game is T, T, T, T, T. Is your friend using a fair coin?*

Solution. The first question to ask is if five tails in a row are possible with a fair coin, that is, with a coin such that $P(T) = P(H) = 0.5$. Indeed,

$$\begin{aligned} P(TTTTT) &= P(T)^5 \qquad \text{(because the coin flips are independent)} \\ &= \left(\frac{1}{2}\right)^5 \approx 0.031 \end{aligned}$$

Then, obtaining five tails in a row is unusual, but it is not impossible. Hence, there are two possibilities:

1. Your friend is very lucky, that is, the coin is fair
2. Your friend is using a coin such that $P(T) > 0.5$

In this case, we would like to test the hypothesis "your friend is lucky" and the collected evidence is the outcome of five flips. In the rest of this chapter we will learn how to analyze this data to conclude if the hypothesis is true or false. □

Similarly to confidence intervals, we will only use data from the sample to evaluate our hypotheses. Then, we cannot be completely sure that our conclusion is correct. We can only determine if the sample data supports the hypothesis or not.

Observe that in the example above, we wrote two statements. The first one is what we want to prove, that is, if the coin is fair; and the second one is what we expect to observe if the coin is not fair. These two statements have a name, and we define them below.

Definition 10.2.

(i) The null hypothesis, denoted H_0, is a statement to be tested and is assumed to be true until evidence indicates otherwise.

(ii) The alternative hypothesis, denoted H_1 is a statement that we are trying to find evidence to support.

From the definition, observe that we assume that H_0 is true, and hypothesis testing tries to show that H_0 is false (and H_1 is true).

Definition 10.3. *There are three possible set ups for null and alternative hypotheses:*

(1) *Two-tailed test:*

$$H_0: \text{ parameter} = \text{value}$$
$$H_1: \text{ parameter} \neq \text{value}$$

(2) *Left-tailed test:*

$$H_0: \text{ parameter} = \text{value}$$
$$H_1: \text{ parameter} < \text{value}$$

(3) *Right-tailed test:*

$$H_0: \text{ parameter} = \text{value}$$
$$H_1: \text{ parameter} > \text{value}$$

Observe that in the three cases, the null hypothesis is the same, and we only change the alternative hypothesis.

Example 10.2. *Identify the null and alternative hypothesis in Example 10.1*

Solution. If we use p to denote the probability of obtaining tails, the null hypothesis is

$$H_0: p = \frac{1}{2}$$

and the alternative hypothesis is

$$H_1: p > \frac{1}{2}$$

□

Let's see another example.

Example 10.3. *Determine the null and the alternative hypothesis in the following situations. Identify if the test is two-sided, left-tailed or right-tailed.*

(a) *The Medco pharmaceutical company has just developed a new antibiotic for children. Two percent of children taking competing antibiotics experience headaches as a side effect. A researcher for the Food and Drug Administration wishes to know if the percentage of children taking the new antibiotic who experience headaches as a side effect is more than 2%.*

(b) *The Blue Book price of a used three-year-old Chevy Corvette ZR1 is $86,012. Grant wonders if the mean price of a used three-year-old Chevy Corvette ZR1 in the Miami metropolitan area is different from $86,012*

Solution.

(a) We are interested in studying whether the population proportion is larger than 2%. Then, the null and alternative hypotheses are:

$$H_0: p = 0.02$$
$$H_1: p > 0.02$$

Then, the test is right-tailed

(b) We are interested in testing whether the average is different from 86,012. Then, the null and alternative hypotheses are:

$$H_0: \overline{x} = 86,012$$
$$H_1: \overline{x} \neq 86,012$$

Then, the test is two-tailed.

□

We have to be very careful when we write the conclusion of our test because we are not able to prove that H_0 is true. Since hypothesis testing seeks evidence that H_1 is true, we can only reject or not reject H_0, that is, we can only say that H_0 is false or that there is not enough evidence to say it is false. You can think of this conclusion as a jury's verdict: a person in a trial can be guilty or not guilty, but never innocent.

As stated before, our conclusion from hypothesis testing can be incorrect because we are using a sample and, hence, incomplete data. The result from our hypothesis test can be rejecting or not rejecting the null hypothesis. However, in reality the alternative hypothesis can be true or false. Hence, there are four possible outcomes from the test. When we make a mistake, we call the error type I or II depending on what was our mistake. We show all the possible outcomes and these two errors in the following table.

		Reality	
		H_0 is true	H_1 is true
Conclusion	Do not reject H_0	Correct conclusion	Type II Error
	Reject H_0	Type I Error	Correct conclusion

Notice that we are using data to determine if the alternative hypothesis is true. However, we cannot know if the null hypothesis is true. We can only say whether there is enough evidence to reject H_0 or not because we are working with a sample.

Example 10.4. *Consider Example 10.3 part (a). Explain what it would mean to make a type I and type II error.*

Solution. Recall that the null and alternative hypotheses are:

$$H_0 : \; p = 0.02$$
$$H_1 : \; p > 0.02$$

According to the table above:

- Type I error is when we reject H_0 even though it is true. In the example, type I error would be if the data shows that $p > 0.02$ but, in reality, the proportion of children who experience a headache is not greater than 2%.
- Type II error is when we do not reject H_0 even though H_1 is true. In the example, type II error would be to conclude that $p = 0.02$ when, in reality, $p > 0.02$.

□

Since our conclusions are based on a sample, there is a probability of making type I and type II errors. The standard notation of these probabilities is

$$\alpha = P(\text{type I error})$$
$$\beta = P(\text{type II error})$$

Notice that if α is very small, there is a high probability that we will not reject H_0. Then, β increases. Hence, there is a trade-off between the probability of type-I and type-II errors.

It is not an accident that we use the same Greek letter to denote the confidence level in a confidence interval, and the probability of type-I error. In both cases, α represents how confident we are in the conclusions we are drawing. In hypothesis testing, α is chosen by the researcher before collecting data and it is called level of significance.

10.2 Hypothesis tests for a population proportion

As discussed before, we always assume that H_0 is true unless we prove otherwise. Assuming that H_0 is true means that the proportion parameter p equals some value, that we denote p_0. Since we only have one random sample as evidence, we judge whether our sample is unusual under the assumption that $p = p_0$. We formally define this concept below.

Definition 10.4. *When observed results are unlikely under the assumption that the null hypothesis is true, we say the result is statistically significant and we reject the null hypothesis.*

In other words, in hypothesis testing we determine if the sample proportion is statistically significant or not.

We will learn two methods to perform hypothesis testing for the population proportion. The first method uses confidence intervals, and is only useful for two-tailed tests. The second method is the classical approach, and is useful for any type of hypothesis.

Hypothesis tests for a population proportion using confidence intervals

Hypothesis testing with confidence intervals is the simplest method. However, it is only valid for two-tailed hypotheses. Specifically, suppose we want to test whether the population proportion equals a specific value p_0 or not, that is, we have

$$H_0 : p = p_0$$
$$H_1 : p \neq p_0$$

The value p_0 is the population proportion we want to test, not the sample proportion. After collecting data and computing the point estimate $\hat{p}$, we can construct the $(1-\alpha)100\%$ confidence interval

$$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Conclusion:

- If p_0 is part of the confidence interval, we conclude that there is not enough evidence to reject H_0
- If p_0 is not part of the confidence interval, we reject H_0 with level of significance α, that is, we conclude that $p \neq p_0$ with level of significance α.

Let's do an example.

Example 10.5. *A study in 2009 found that 34% of teenagers text while driving. A recent survey conducted to 1200 teenagers found that 353 of them had texted while driving. Does the recent survey suggest that the proportion of teens who text while driving has changed since 2009? Use level of significance 0.05.*

Solution. The study want to find out whether the proportion of teens who drive equals 34% or not. Then, the null and alternative hypotheses are

$$H_0 : p = 0.34$$
$$H_1 : p \neq 0.34$$

Then, $p_0 = 0.34$.

Using the recent data, we can construct a confidence interval about the population proportion. For a level of significance $\alpha = 0.05$, we construct a 95% confidence interval. We have

$$\hat{p} = \frac{x}{n} = \frac{353}{1200} = 0.2942$$

and we obtain the confidence interval

$$\begin{aligned}\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= 0.2942 \pm 1.96\sqrt{\frac{0.2942 \cdot 0.7058}{1200}} \\ &= 0.2942 \pm 0.0258\end{aligned}$$

Then, the lower bound of the confidence interval is 0.2684 and the upper bound is 0.32. The value $p_0 = 0.34$ is greater than the upper bound 0.32. Hence, we reject H_0 and conclude that the proportion of teens who text while driving has changed with 5% level of significance. □

Hypothesis tests about the population proportion using the classical approach

In this method, we evaluate if the sample is statistically significant (or sufficient) by checking whether the sample proportion $\hat{p}$ is far away from the population proportion. To answer this question, we use that the sampling distribution of the sample proportion $\hat{p}$ is normal with mean and standard deviation depending on the population proportion p. Specifically, we have

$$\mu_{\hat{p}} = p \qquad \text{and} \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Then, we can compute the z-score of the population proportion $\hat{p}$ to determine how many standard deviation away from the mean its value is. The notion of "too far away" will be determined by the level of significance α.

This method is valid for all three types of alternative hypotheses, and we specify the steps below.

(0) Check if $np_0(1-p_0) \geq 10$ and that the sample size n is less than 5% of the population size

(1) Determine the null and alternative hypotheses. They can be:

- (i) Two tailed: $H_0 : p = p_0$, $H_1 : p \neq p_0$
- (ii) Left tailed: $H_0 : p = p_0$, $H_1 : p < p_0$
- (iii) Right tailed: $H_0 : p = p_0$, $H_1 : p > p_0$

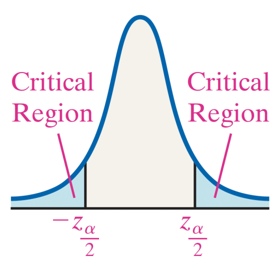
(2) Select the level of significance α.

(3) Compute the test statistic

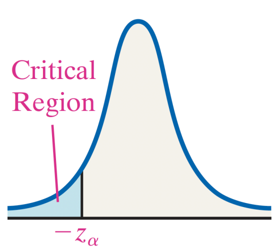
$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$$

(4) Compute the critical value(s) using the last row of the t-distribution table and compare to the test statistic, as follows:

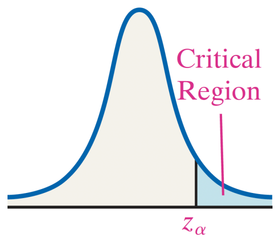
(i) Two-tailed:
The critical values are $-z_{\frac{\alpha}{2}}$ and $z_{\frac{\alpha}{2}}$, and we reject H_0 if $z_0 < -z_{\frac{\alpha}{2}}$ or $z_0 > z_{\frac{\alpha}{2}}$. Pictorially, we reject H_0 if z_0 is part of the tails highlighted in blue in the figure:



(ii) Left-tailed:
The critical value is $-z_\alpha$, and we reject H_0 if $z_0 < -z_\alpha$. Pictorially, we reject H_0 if z_0 is part of the tail highlighted in blue in the figure:



(iii) Right-tailed:
The critical value is z_α, and we reject H_0 if $z_0 > z_\alpha$. Pictorially, we reject H_0 if z_0 is part of the tail highlighted in blue in the figure:



Observe that the critical regions respond to the alternative hypothesis (H_1) being unusual if we assume that the null hypothesis (H_0) is true. Let's see an example.

Example 10.6. *The two major college entrance exams that a majority of colleges accept for admission are the SAT and ACT. ACT looked at historical records and established 22 as the minimum ACT math score for a student to be considered prepared for college mathematics.*

An official with the Illinois State Department of Education wonders whether less than half of the students in her state are prepared for college mathematics. She obtains a simple random sample of 500 records of students who have taken the ACT and finds that 219 are prepared for college mathematics. Does this represent significant evidence that less than half of Illinois students who have taken the ACT are prepared for college mathematics upon graduation of calculus? Use $\alpha = 0.05$ level of confidence.

Solution. From the text, we extract that $p_0 = 0.5$ because we want to see if less than half of the students in the study are prepared. Also, from the sample we obtain

$$\hat{p} = \frac{219}{500} = 0.438$$

Now we follow the steps specified above. We have:

(0) We verify the conditions. Observe that $n = 500$ is probably well below 5% of college students in Illinois because 500 is 5% of 10000, and there are clearly more than 10000 college students in Illinois.

Additionally,

$$np_0(1 - p_0) = 500 \cdot 0.5 \cdot 0.5 = 125 \geq 10$$

Hence, the conditions are satisfied.

(1) The null and alternative hypotheses are:

$$H_0 : p = 0.5$$
$$H_1 : p < 0.5$$

Hence, our hypothesis is left-tailed.

(2) The significance level $\alpha = 0.05$ is given.

(3) We compute the statistic z_0 as follows:

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.438 - 0.5}{\sqrt{\frac{0.5 \cdot 0.5}{500}}} = -2.77$$

(4) Since this is a left-tailed test, we compute the critical value $-z_\alpha$ from the t-table as follows:

Table VII

Degrees of Freedom	*t*-Distribution Area in Right Tail											
	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.894	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291

and obtain

$$-z_\alpha = -z_{0.05} = -1.645$$

Since

$$z_0 = -2.77 < -z_\alpha = -1.645$$

we reject H_0 and conclude that there is enough evidence to say that less than half of Illinois students are prepared for college-level mathematics.

□

Now let's see a two-tailed example.

Example 10.7. *Consider again Example 10.5. Use the classical approach to decide if the recent survey suggests that the proportion of teens who text while driving has changed since 2009. Use level of significance* $\alpha = 0.05$.

Solution. Recall that we had:

$$p_0 = 0.5, \qquad \hat{p} = 0.2942, \qquad n = 1200.$$

Similarly to the previous example, we follow the steps. We obtain the following:

(0) We can assume that 1200 is less than 5% of the population, and observe

$$np_0(1 - p_0) = 1200 \cdot 0.5 \cdot 0.5 = 300 \geq 10$$

Then, the conditions are satisfied.

(1) The null and alternative hypotheses are:

$$H_0 : p_0 = 0.5$$
$$H_1 : p_0 \neq 0.5$$

Then, we run a two-tailed test.

(2) The significance level is $\alpha = 0.05$

(3) The test statistic is

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.2942 - 0.5}{\sqrt{\frac{0.5 \cdot 0.5}{1200}}} = -14.2582$$

(4) Since the test is two-tailed, the critical values are $-z_{\frac{\alpha}{2}}$ and $z_{\frac{\alpha}{2}}$. We use the t-table to search for the critical value, as shown in the figure: and obtain

Table VII

Degrees of Freedom	*t*-Distribution Area in Right Tail 0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.894	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291

$$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$$

Then, since $p_0 < -z_{\frac{\alpha}{2}}$, we reject H_0. That is, we conclude that there is enough evidence to say that the proportion of teens who text while driving has changed since 2009 with 95% confidence.

□

10.3 Hypothesis tests for a population mean

We use the same ideas developed for the population proportion, but now for the sample mean. The only difference is that we use the t distribution instead of the normal distribution.

Recall that we use μ to denote the population mean. Then, we will use μ_0 for the value we want to test. That is, the null hypothesis is $H_0 : \mu = \mu_0$.

Hypothesis tests for a population mean using confidence intervals

Similarly to the population proportion, we can only use this test for two-tailed hypotheses, that is, when the null and alternative hypotheses are:

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

After collecting the data, we can construct the $(1 - \alpha)100\%$ confidence interval as

$$\overline{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

Conclusion:

- If μ_0 is part of the confidence interval, we do not reject H_0
- If μ_0 is not part of the confidence interval, we reject H_0 with level of significance α, that is, we conclude that $\mu \neq \mu_0$ with level of significance α.

Let's do an example.

Example 10.8. *In 2001, the mean household expenditure for energy was $1493, according to data from the U.S. Energy Information Administration. An economist wanted to know whether this amount has changed significantly from its 2001 level.*

In a random sample of 35 households, he found the mean expenditure (in 2001 dollars) for energy during the most recent year to be $1618, with a standard deviation $321. Conduct a hypothesis test to determine if the mean expenditure is different from the mean expenditure in 2001. Use level of significance $\alpha = 0.01$.

Solution. In this case, $\mu_0 = 1493$ and out hypotheses are:

$$H_0 : \mu = 1493$$
$$H_1 : \mu \neq 1493$$

We construct a 99% confidence interval. We know

$$\overline{x} = 1618, \qquad s = 321, \qquad n = 35.$$

and we obtain $t_{\frac{\alpha}{2}}$ from the t-table, using $n - 1 = 34$ degrees of freedom. From the table, we have:

Table VII

***t*-Distribution**
Area in Right Tail

Degrees of Freedom	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.894	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
31	0.682	0.853	1.054	1.309	1.696	2.040	2.144	2.453	2.744	3.022	3.375	3.633
32	0.682	0.853	1.054	1.309	1.694	2.037	2.141	2.449	2.738	3.015	3.365	3.622
33	0.682	0.853	1.053	1.308	1.692	2.035	2.138	2.445	2.733	3.008	3.356	3.611
34	0.682	0.852	1.052	1.307	1.691	2.032	2.136	2.441	2.728	3.002	3.348	3.601
35	0.682	0.852	1.052	1.306	1.690	2.030	2.133	2.438	2.724	2.996	3.340	3.591

That is, $t_{\frac{\alpha}{2}} = 2.728$.
Therefore, the confidence interval is

$$\begin{aligned} \overline{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} &= 1618 \pm 2.728 \cdot \frac{321}{\sqrt{35}} \\ &= 1618 \pm 148.018 \end{aligned}$$

Then, the lower bound of the confidence interval is 1469.982 and the upper bound is 1766.018. Since $\mu_0 = 1493$ is within these values, we do not reject H_0. □

Hypothesis tests about the population mean using the classical approach

The steps are detailed below:

(0) Check that the following conditions are satisfied:

- The sample has no outliers
- The sample size satisfied $n \geq 30$, or it comes from a normally distributed population.
- The sample size n is less than 5% of the population size

(1) Determine the null and alternative hypotheses. They can be:

(i) Two tailed: $H_0 : \mu = \mu_0$, $H_1 : \mu \neq \mu_0$
(ii) Left tailed: $H_0 : \mu = \mu_0$, $H_1 : \mu < \mu_0$
(iii) Right tailed: $H_0 : \mu = \mu_0$, $H_1 : \mu > \mu_0$

(2) Select the level of significance α

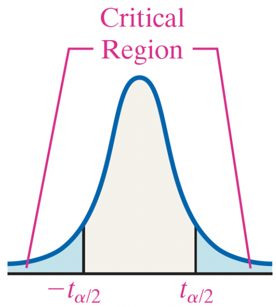
(3) Compute the test statistic:

$$t_0 = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

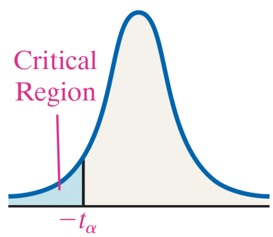
and recall t_0 follows a t distribution with $n - 1$ degrees of freedom.

(4) Compute the critical value(s) using the t-distribution table and compare it to the test statistic, as follows:

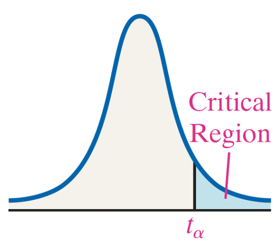
(i) Two tailed:
The critical values are $-t_{\frac{\alpha}{2}}$ and $t_{\frac{\alpha}{2}}$, and we reject H_0 if $t_0 < -t_{\frac{\alpha}{2}}$ or if $t_0 > t_{\frac{\alpha}{2}}$. Pictorially, we reject H_0 if t_0 is part of the blue tails in the figure:



(ii) Left-tailed:
The critical value is $-t_\alpha$, and we reject H_0 if $t_0 < -t_\alpha$. Pictorially, we reject H_0 if t_0 is part of the blue tail in the figure:



(iii) Right-tailed:
The critical value is t_α, and we reject H_0 if $t_0 > t_\alpha$. Pictorially, we reject H_0 if t_0 is part of the blue tail in the figure:



Let's do an example.

Example 10.9. *The mean height of American males is 69.5 inches. The heights of the 43 male U.S. presidents up to 2013 (Washington through Obama) have a mean 70.78 inches, a standard deviation of 2.77 inches and no outliers. Treating the 43 presidents as a simple random sample, determine if there is evidence to suggest that U.S. presidents are taller than the average American male. Use the* $\alpha = 0.05$ *level of significance.*

Solution. We follow the steps specified above:

(0) We first verify the conditions. We are told that there are no outliers, the sample size is $n = 43 \geq 30$, and the population of male Americans is roughly half of the US population, which is 300 million people. Hence, $n = 43$ is well below 5%.

(1) We want to test if the U.S. presidents are taller than the average American, whose height is $\mu_0 = 69.5$ inches. Then,

$$H_0 : \mu = 69.5$$
$$H_1 : \mu > 69.5$$

Then, we run a right-tailed test.

(2) The level of significance is given: $\alpha = 0.05$

(3) To compute the test statistic, observe that the sample is the set of presidents. Then, $\overline{x} = 70.78$ and $s = 2.77$. Therefore, the test statistic is:

$$t_0 = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{70.78 - 69.5}{\frac{2.77}{\sqrt{43}}} = 3.030$$

(4) This is a right-tailed test, so the critical value is $t_\alpha = t_{0.05}$ and has 42 degrees of freedom. We look at the t-table and try to find the row corresponding to $n - 1 = 42$ degrees of freedom. We don't have exactly that line, so we use the closest available, that is, we use 40 degrees of freedom, as shown in the figure:

Table VII

***t*-Distribution**
Area in Right Tail

Degrees of Freedom	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.894	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
38	0.681	0.851	1.051	1.304	1.686	2.024	2.127	2.429	2.712	2.980	3.319	3.566
39	0.681	0.851	1.050	1.304	1.685	2.023	2.125	2.426	2.708	2.976	3.313	3.558
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
70	0.678	0.847	1.044	1.294	1.667	1.994	2.093	2.381	2.648	2.899	3.211	3.435
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
90	0.677	0.846	1.042	1.291	1.662	1.987	2.084	2.368	2.632	2.878	3.183	3.402
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291

That is, $t_\alpha = 1.684$. Then, we have

$$t_0 = 3.030 > t_\alpha = 1.684.$$

Hence, we reject H_0.

$\square$